

हमारा विश्वास... हर एक विद्यार्थी है खास

**JEE
MAIN
Sept.
2020**

QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 2 Sep. _ SHIFT - 2



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Q.1 Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function which satisfies $f(x+y) = f(x) + f(y) \forall x, y \in \mathbb{R}$. If $f(1) = 2$ and

$$g(n) = \sum_{k=1}^{(n-1)} f(k), n \in \mathbb{N} \text{ then the value of } n, \text{ for which } g(n) = 20, \text{ is:}$$

माना एक फलन $f : \mathbb{R} \rightarrow \mathbb{R}$, प्रत्येक $x, y \in \mathbb{R}$ के लिए $f(x+y) = f(x) + f(y)$ संतुष्ट करता है। यदि $f(1) = 2$ तथा

$$g(n) = \sum_{k=1}^{(n-1)} f(k), n \in \mathbb{N} \text{ है, तो } n \text{ का वह मान जिसके लिए } g(n) = 20 \text{ है, है:}$$

(1) 9

(2) 5

(3) 4

(4) 20

Sol.

(2)

$$f(1) = 2 ; f(x+y) = f(x) + f(y)$$

$$x = y = 1 \Rightarrow f(2) = 2 + 2 = 4$$

$$x = 2, y = 1 \Rightarrow f(3) = 4 + 2 = 6$$

$$g(n) = f(1) + f(2) + \dots + f(n-1)$$

$$= 2 + 4 + 6 + \dots + 2(n-1)$$

$$= 2 \sum (n-1)$$

$$= 2 \frac{(n-1) \cdot n}{2}$$

$$= n^2 - n$$

$$\text{Given } g(n) = 20 \Rightarrow n^2 - n = 20$$

$$n^2 - n - 20 = 0$$

$$n = 5$$

Q.2 If the sum of first 11 terms of an A.P., a_1, a_2, a_3, \dots is $0 (a_1 \neq 0)$ then the sum of the A.P., $a_1, a_3, a_5, \dots, a_{23}$ is ka_1 , where k is equal to:

यदि A.P., a_1, a_2, a_3, \dots के प्रथम 11 पदों का योगदान $0 (a_1 \neq 0)$ है, और A.P., $a_1, a_3, a_5, \dots, a_{23}$ का योगफल ka_1 है, तो k बराबर है:

(1) $-\frac{121}{10}$

(2) $-\frac{72}{5}$

(3) $\frac{72}{5}$

(4) $\frac{121}{10}$

Sol.

(2)

$$\sum_{k=1}^{11} a_k = 0 \Rightarrow 11a + 55d = 0$$

$$a + 5d = 0$$

$$\text{Now } a_1 + a_3 + \dots + a_{23} = ka_1$$

$$12a + d(2+4+6+\dots+22) = ka$$

$$12a + 2d \cdot 66 = ka$$

$$12(a+11d) = ka$$

$$12 \left(a + 11 \left(-\frac{a}{5} \right) \right) = ka$$

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$$12\left(1 - \frac{11}{5}\right) = k$$

$$k = -\frac{72}{5}$$

Q.3 Let E^c denote the complement of an event E . Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$. Then $P(E_2^c \cap E_3^c / E_1)$ is equal to:

माना E^c घटना E का पूरक है। यदि कोई तीन घटनाएँ E_1, E_2 तथा E_3 युग्मों में स्वतंत्र हैं, तथा $P(E_1) > 0$ और $P(E_1 \cap E_2 \cap E_3) = 0$,

तो $P(E_2^c \cap E_3^c / E_1)$ बराबर है:

(1) $P(E_3^c) - P(E_2^c)$

(2) $P(E_3) - P(E_2^c)$

(3) $P(E_3^c) - P(E_2)$

(4) $P(E_2^c) + P(E_3)$

Sol. (3)

$$\begin{aligned} P(E_2^c \cap E_3^c / E_1) &= \frac{P(E_2^c \cap E_3^c \cap E_1)}{P(E_1)} \\ &= \frac{P(E_1) - P(E_1 \cap E_2) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3)}{P(E_1)} \\ &= \frac{P(E_1) - P(E_1) \cdot P(E_2) - P(E_1) \cdot P(E_3) + 0}{P(E_1)} = 1 - P(E_2) - P(E_3) \\ &= P(E_3^c) - P(E_2) \end{aligned}$$

Q.4 If the equation $\cos^4 \theta + \sin^4 \theta + \lambda = 0$ has real solutions for θ , then λ lies in the interval:

यदि समीकरण $\cos^4 \theta + \sin^4 \theta + \lambda = 0$ के θ में वास्तविक हल हैं, तो λ निम्न में से किस अन्तराल में स्थित है?

(1) $\left[-\frac{1}{2}, -\frac{1}{4}\right]$

(2) $\left[-1, -\frac{1}{2}\right]$

(3) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$

(4) $\left[-\frac{5}{4}, -1\right]$

Sol. (2)

$$\cos^4 \theta + \sin^4 \theta + \lambda = 0$$

$$\lambda = -\left\{1 - \frac{1}{2} \sin^2 2\theta\right\}$$

$$2(\lambda + 1) = \sin^2 2\theta$$

$$0 \leq 2(\lambda + 1) \leq 1$$

$$0 \leq \lambda + 1 \leq \frac{1}{2}$$

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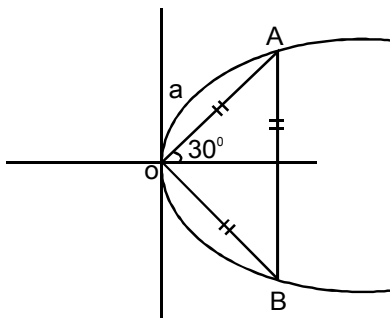
$$\boxed{-1 \leq \lambda \leq -\frac{1}{2}}$$

Q.5 The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is:

एक समबाहु त्रिभुज, जिसका एक शीर्ष, परवलय $y^2 = 8x$ के शीर्ष पर है, परवलय के अंतर्गत खींचा गया है। तो त्रिभुज का क्षेत्रफल (वर्ग इकाईयों में) है:

- (1) $128\sqrt{3}$ (2) $192\sqrt{3}$ (3) $64\sqrt{3}$ (4) $256\sqrt{3}$

Sol. (2)



A : $(a \cos 30^\circ, a \sin 30^\circ)$
lies on parabola

$$\frac{a^2}{4} = 8 \cdot \frac{a\sqrt{3}}{2}$$

$$\boxed{a = 16\sqrt{3}}$$

$$\text{Area of equilateral } \Delta = \frac{\sqrt{3}}{4} a^2$$

$$\Delta = \frac{\sqrt{3}}{4} \cdot 16 \cdot 16 \cdot 3$$

$$\Delta = 192\sqrt{3}$$

Q.6 The imaginary part of $(3 + 2\sqrt{-54})^{1/2} - (3 - 2\sqrt{-54})^{1/2}$ can be :

$(3 + 2\sqrt{-54})^{1/2} - (3 - 2\sqrt{-54})^{1/2}$ काल्पनिक भाग हो सकता है:

- (1) $\sqrt{6}$ (2) $-2\sqrt{6}$ (3) 6 (4) $-\sqrt{6}$

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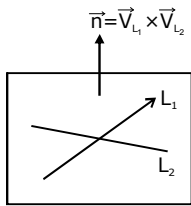
Sol. (2)

$$\begin{aligned} & (3 + 2i\sqrt{54})^{1/2} - (3 - 2i\sqrt{54})^{1/2} \\ &= (9 + 6i^2 + 2 \cdot 3i\sqrt{6})^{1/2} - (9 + 6i^2 - 2 \cdot 3i\sqrt{6})^{1/2} \\ &= \left((3 + \sqrt{6}i)^2 \right)^{1/2} - \left((3 - \sqrt{6}i)^2 \right)^{1/2} \\ &= \pm(3 + \sqrt{6}i) \mp (3 - \sqrt{6}i) = -2\sqrt{6}i \end{aligned}$$

Q.7 A plane passing through the point (3,1,1) contains two lines whose direction ratios are 1, -2, 2 and 2, 3, -1 respectively. If this plane also passes through the point (α , -3, 5), then α is equal to:
 बिन्दु (3,1,1) से होकर जाने वाले समतल में दो सरल रेखाएँ स्थित हैं जिनके दिक् अनुपात (direction ratios) क्रमशः 1, -2, 2 तथा 2, 3, -1 हैं। यदि यह समतल बिन्दु (α , -3, 5) से भी होकर जाता है, तो α बराबर है:

- (1) -5 (2) 10 (3) 5 (4) -10

Sol. (3)



$$\begin{aligned} \vec{n}_p &= (-4, 5, 7) \\ \text{Equation of plane :} \\ P &: -4(x-3) + 5(y-1) + 7(z-1) = 0 \\ P &: -4x + 5y + 7z + 12 - 5 - 7 = 0 \\ P &: 4x - 5y - 7z = 0 \\ \text{Pass } (\alpha, -3, 5) \\ 4\alpha + 15 - 35 &= 0 \\ 4\alpha &= 20 \\ \alpha &= 5 \end{aligned}$$

Q.8 Let $A = \{X = (x, y, z)^T : PX = 0 \text{ and } x^2 + y^2 + z^2 = 1\}$, where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$, then the set A:

- (1) contains more than two elements (2) is a singleton.
 (3) contains exactly two elements (4) is an empty set.

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यदि $A = \{X = (x, y, z)^T : PX = 0 \text{ तथा } x^2 + y^2 + z^2 = 1\}$, जबकि $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$, है, तो समुच्चय A:

- (1) में दो से अधिक अवयव हैं।
 (2) एक ही अवयव वाला समुच्चय है।
 (3) में ठीक दो अवयव विद्यमान हैं।
 (4) एक रिक्त समुच्चय है।

Sol. (3)

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ \& } x^2 + y^2 + z^2 = 1$$

$$PX = 0$$

$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x + 2y + z = 0 \dots\dots\dots(1)$$

$$-2x + 3y - 4z = 0 \dots\dots(2)$$

$$x + 9y - z = 0 \dots\dots\dots(3)$$

from (1) & (3)

$$\Rightarrow 2x + 11y = 0$$

from (1) & (2)

$$\Rightarrow 2x + 11y = 0$$

from (2) & (3)

$$-6x - 33y = 0$$

$$\Rightarrow 2x + 11y = 0$$

put in (1)

$$-7y + 2z = 0$$

$$\text{Now } \left(\frac{11y}{2}\right)^2 + y^2 + \left(\frac{7y}{2}\right)^2 = 1$$

$$y^2(121 + 1 + 49) = 4$$

$$y^2(171) = 4$$

$$y = \pm \frac{2}{\sqrt{171}} \Rightarrow x = \pm \frac{7}{\sqrt{171}} \Rightarrow z = \mp \frac{11}{\sqrt{171}} \Rightarrow \text{Only two pair possible}$$

Q.9 The equation of the normal to the curve $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ at $x=0$ is:

$x = 0$ पर, वक्र $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ पर खींचे गए अभिलम्ब का समीकरण है

- (1) $y+4x=2$ (2) $2y+x=4$ (3) $x+4y=8$ (4) $y=4x+2$

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Sol. (3)

at $x = 0 \Rightarrow y = 1 + \cos^2(0) = 2$

$p : (0, 2)$

Now $y' = (1+x)^{2y} \left\{ \frac{2y}{1+x} + \ln(1+x) \cdot 2y \right\} - \sin 2(\sin^{-1}x) \cdot \frac{1}{\sqrt{1-x^2}}$

$y \Big|_{(0,2)} = 4 - 0$

$N_o : y - 2 = -\frac{1}{4}(x-0)$

$N_o : 4y - 8 = -x$

$N_o : \boxed{x + 4y = 8}$

Q.10 Consider a region $R = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y \leq 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true.?

एक क्षेत्र $R = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y \leq 2x\}$ पर विचार कीजिए। यदि एक सरल रेखा $y = \alpha$, क्षेत्र R के क्षेत्रफल को दो बराबर भागों में बांटती है, तो निम्न में से कौन-सा सत्य है?

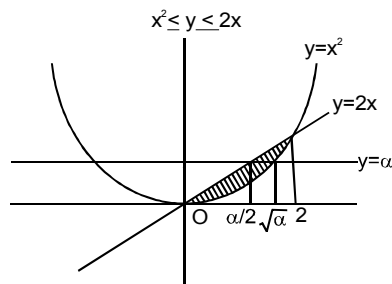
(1) $\alpha^3 - 6\alpha^2 + 16 = 0$

(2) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

(3) $\alpha^3 - 6\alpha^{3/2} - 16$

(4) $3\alpha^2 - 8\alpha + 8 = 0$

Sol. (2)



$A = \text{Area} = \int_0^2 (2x - x^2) dx = 4 - \frac{8}{3} = \boxed{\frac{4}{3}}$

Now $\int_0^{\alpha/2} (2x - x^2) dx + \int_{\alpha/2}^{\sqrt{\alpha}} (\alpha - x^2) dx = \frac{1}{2}A$

$\frac{\alpha^2}{4} - \frac{\alpha^3}{24} + \alpha(\sqrt{\alpha} - \alpha/2) - \left(\frac{\alpha\sqrt{\alpha}}{3} - \frac{\alpha^3}{24} \right) = \frac{4}{6}$

$\frac{\alpha^2}{4} + \alpha\sqrt{\alpha} - \frac{\alpha^2}{2} - \frac{\alpha\sqrt{\alpha}}{3} = \frac{4}{6}$

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$$-3\alpha^2 + 8\alpha\sqrt{\alpha} = 8$$

$$3\alpha^2 - 8\alpha\sqrt{\alpha} + 8 = 0$$

Q.11 Let $f : (-1, \infty) \rightarrow \mathbb{R}$ be defined by $f(0)=1$ and $f(x) = \frac{1}{x} \log_e(1+x), x \neq 0$. Then the function f :

- (1) increases in $(-1, \infty)$
- (2) decreases in $(-1, 0)$ and increases in $(0, \infty)$
- (3) increases in $(-1, 0)$ and decreases in $(0, \infty)$
- (4) decreases in $(-1, \infty)$.

यदि $f : (-1, \infty) \rightarrow \mathbb{R}$ $f(0) = 1$ तथा $f(x) = \frac{1}{x} \log_e(1+x), x \neq 0$ द्वारा परिभाषित है, तो फलन f :

- (1) $(-1, \infty)$ में वर्धमान है।
- (2) $(-1, 0)$ में ह्रासमान है तथा $(0, \infty)$ में वर्धमान है।
- (3) $(-1, 0)$ में वर्धमान है तथा $(0, \infty)$ में ह्रासमान है।
- (4) $(-1, \infty)$ में ह्रासमान है।

Sol. (4)

$$f(x) = \frac{1}{x} \ln(1+x)$$

$$f' = \frac{x - \frac{1}{1+x} - \ln(1+x)}{x^2}$$

$$f' = \frac{1 - \frac{1}{1+x} - \ln(1+x)}{x^2}$$

$$f' < 0 \quad \forall x \in (-1, \infty)$$

Q.12 Which of the following is a tautology?

निम्न में से कौन-सा कथन पुनरुक्ति है?

- (1) $(p \rightarrow q) \wedge (q \rightarrow p)$
- (2) $(\sim p) \wedge (p \vee q) \rightarrow q$
- (3) $(q \rightarrow p) \vee \sim(p \rightarrow q)$
- (4) $(\sim q) \vee (p \wedge q) \rightarrow q$

Sol. (2)

$\sim p$	$p \vee q$	$\sim p \wedge (p \vee q)$	$\sim p \wedge (p \vee q) \rightarrow q$
F	T	F	T
F	T	F	T
T	T	T	T
T	F	F	T

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Q.13 Let $f(x)$ be a quadratic polynomial such that $f(-1)+f(2)=0$. If one of the roots of $f(x)=0$ is 3, then its other roots lies in:

माना $f(x)$ एक द्विघात बहुपद है जिसके लिए $f(-1)+f(2)=0$ है। यदि $f(x)=0$ का एक मूल 3 है, तो दूसरा मूल निम्न में से किस अन्तराल में स्थित है?

- (1) (0,1) (2) (1,3) (3) (-1,0) (4) (-3,-1)

Sol. (3)

Let $f(x) = a(x-3)(x-\alpha)$
 $f(-1)+f(2)=0$
 $a[(-1-3)(-1-\alpha)+(2-3)(2-\alpha)]=0$
 $a[4+4\alpha-2+\alpha]=0$
 $5\alpha+2=0$

$$\alpha = -\frac{2}{5}$$

Q.14 Let S be the sum of the first 9 terms of the series :

$\{x+ka\} + \{x^2+(k+2)a\} + \{x^3+(k+4)a\} + \{x^4+(k+6)a\} + \dots$ where $a \neq 0$ and $a \neq 1$.

If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to:

माना S, श्रेणी $\{x+ka\} + \{x^2+(k+2)a\} + \{x^3+(k+4)a\} + \{x^4+(k+6)a\} + \dots$ जहाँ $a \neq 0$ तथा $x \neq 1$ है, के प्रथम 9 पदों

का योगफल है। यदि $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$ है, तो k बराबर है:

- (1) 3 (2) -3 (3) 1 (4) -5

Sol. (2)

$S = \{x+ka\} + \{x^2+(k+2)a\} + \{x^3+(k+4)a\}$ up to 9 term
 $S = (x+x^2+\dots+x^9)+a\{k+(k+2)+(k+4)+\dots\}$ up to 9 term

$$S = \frac{x(1-x^9)}{1-x} + a\{9k+2.36\}$$

$$S = \frac{x^{10} - x}{x-1} + 9ak + 72a$$

$$S = \frac{x^{10} - x + 45a(x-1)}{x-1} = \frac{x^{10} - x + (9k+72)a(x-1)}{x-1}$$

$$= 45 = 9k + 72$$

$$9k = -27$$

$$k = -3$$

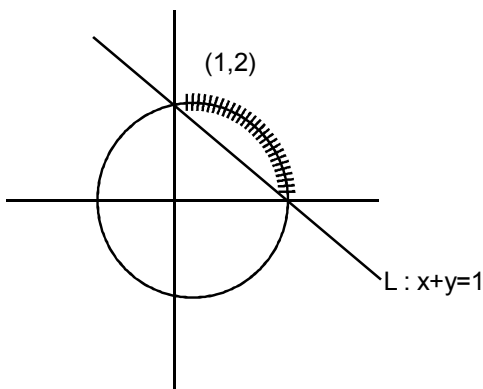
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Q.15 The set of all possible values of θ in the interval $(0, \pi)$ for which the points $(1, 2)$ and $(\sin \theta, \cos \theta)$ lie on the same side of the line $x + y = 1$ is:

अन्तराल $(0, \pi)$ में θ के सभी संभावित मूल्यों का समुच्चय, जिसके लिए दोनों बिन्दु, $(1, 2)$ तथा $(\sin \theta, \cos \theta)$, सरल रेखा $x + y = 1$ के एक ही तरफ स्थित हैं, है:

- (1) $\left(0, \frac{\pi}{4}\right)$ (2) $\left(0, \frac{\pi}{2}\right)$ (3) $\left(0, \frac{3\pi}{4}\right)$ (4) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

Sol. (2)



$(\sin \theta, \cos \theta)$ lie on $x^2 + y^2 = 1$

Shaded points satisfy

$$\Rightarrow \theta \in (0, \pi / 2)$$

Q.16 Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is:

माना $n > 2$ एक पूर्णांक है तथा एक शहर में n मेट्रो स्टेशन हैं जो एक वृत्ताकार पथ पर स्थित हैं। प्रत्येक दो स्टेशन एक सीधे ट्रैक (Track) से जोड़े गए हैं। इसके अतिरिक्त, प्रत्येक दो निकटतम स्टेशन ब्लू लाईन्स (Blue Line) से तथा अन्य सभी दो स्टेशन रेड लाईन्स (Red Line) से जोड़े गए हैं। यदि रेड लाईन्स की संख्या ब्लू लाईन्स की संख्या का 99 गुणा है, तो n का मान है:

- (1) 201 (2) 199 (3) 101 (4) 200

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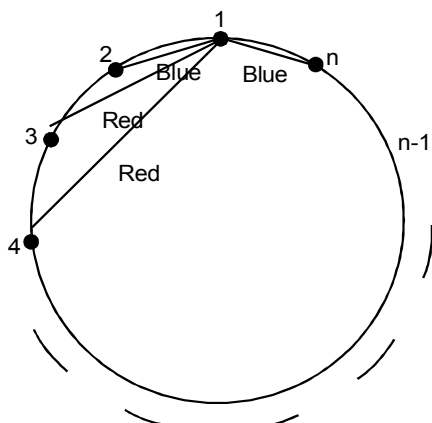
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Sol. (1)



Red line = 99 blue line

$${}^n C_2 - n = 99n$$

$$\frac{n(n-1)}{2} = 100n$$

$$n-1 = 200$$

$$\boxed{n = 201}$$

Q.17 If a curve $y=f(x)$, passing through the point $(1,2)$ is the solution of the differential equation, $2x^2 dy = (2xy + y^2) dx$, then $f\left(\frac{1}{2}\right)$ is equal to:

यदि एक वक्र, $y=f(x)$, बिन्दु $(1,2)$ से होकर जाता है तथा अवकल समीकरण $2x^2 dy = (2xy + y^2) dx$, का हल है, तो $f\left(\frac{1}{2}\right)$ बराबर है:

(1) $\frac{-1}{1 + \log_e 2}$

(2) $1 + \log_e 2$

(3) $\frac{1}{1 + \log_e 2}$

(4) $\frac{1}{1 - \log_e 2}$

Sol. (3)

$$2 \frac{dy}{dx} = 2 \frac{y}{x} + \left(\frac{y}{x}\right)^2 \rightarrow \text{HDE}$$

$$\therefore y = vx$$

$$2 \left(v + x \frac{dv}{dx} \right) = 2v + v^2$$

$$2 \frac{dv}{v^2} = \frac{dx}{x}$$

$$-\frac{2}{v} = \ln x + c$$

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$$-\frac{2x}{y} = \ln x + c$$

$$\downarrow (1,2)$$

$$c = -1$$

$$c : \ln x + \frac{2x}{y} = 1$$

$$\text{For } f(1/2) \Rightarrow \ln\left(\frac{1}{2}\right) + \frac{2}{2y} = 1$$

$$y = \frac{1}{1 + \ln 2}$$

Q.18 For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2 \sec^2 \theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2 \sec^2 \theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is:

किसी $\theta \in \left(0, \frac{\pi}{2}\right)$ के लिए, यदि अतिपरवलय, $x^2 - y^2 \sec^2 \theta = 10$ की उत्केन्द्रता, दीर्घवृत्त, $x^2 \sec^2 \theta + y^2 = 5$ की उत्केन्द्रता

का $\sqrt{5}$ गुणा है, तो दीर्घवृत्त की नाभिलम्ब जीवा की लम्बाई बराबर है:

(1) $\frac{4\sqrt{5}}{3}$

(2) $\frac{2\sqrt{5}}{3}$

(3) $2\sqrt{6}$

(4) $\sqrt{30}$

Sol. (1)

$$H : x^2 - y^2 \sec^2 \theta = 10$$

$$E : x^2 \sec^2 \theta + y^2 = 5$$

$$\sqrt{1 + \frac{10 \cos^2 \theta}{10}} = \sqrt{5} \sqrt{1 - \frac{5 \cos^2 \theta}{5}}$$

$$1 + \cos^2 \theta = 5 - 5 \cos^2 \theta$$

$$6 \cos^2 \theta = 4$$

$$\cos \theta = \pm \sqrt{\frac{2}{3}}$$

$$l(\text{LR}) \text{ of ellipse} = \frac{2.5 \cos^2 \theta}{\sqrt{5}}$$

$$= 2\sqrt{5} \cdot \frac{2}{3} = \boxed{\frac{4\sqrt{5}}{3}}$$

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Q.19 $\lim_{x \rightarrow 0} \left(\tan \left(\frac{\pi}{4} + x \right) \right)^{1/x}$ is equal to:

$\lim_{x \rightarrow 0} \left(\tan \left(\frac{\pi}{4} + x \right) \right)^{1/x}$ बराबर है:

(1) e

(2) e²

(3) 2

(4) 1

Sol.

$\lim_{x \rightarrow 0} \left(\tan \left(\frac{\pi}{4} + x \right) \right)^{1/x} (1^\infty) = e^L$

$$L = \lim_{x \rightarrow 0} \frac{\tan \left(\frac{\pi}{4} + x \right) - 1}{x}$$

$$L = \lim_{x \rightarrow 0} \frac{1 + \tan x - 1}{1 - \tan x - x}$$

$$L = \lim_{x \rightarrow 0} 2 \left(\frac{\tan x}{x} \right) \cdot \left(\frac{1}{1 - \tan x} \right)$$

L = +2
Ans. e²

Q.20 Let a, b, c ∈ R be all non-zero and satisfy a³+b³+c³ = 2. If the matrix

$$A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

satisfies A^TA=I, then a value of abc can be:

माना a, b, c ∈ R सभी अशून्य हैं और a³+b³+c³ = 2 को संतुष्ट करते हैं। यदि आव्यूह $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ के लिए A^TA=I, है,

तो abc का एक मान हो सकता है:

(1) $\frac{2}{3}$

(2) 3

(3) $-\frac{1}{3}$

(4) $\frac{1}{3}$

Sol.

(4)
a³+b³+c³=2
A^TA = I

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$$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= a^2 + b^2 + c^2 = 1$$

$$\& ab + bc + ca = 0$$

$$\text{Now } (a+b+c)^2 = \sum a^2 + 2\sum ab$$

$$(\sum a)^2 = 1 + 0 \Rightarrow (\sum a)^2 = 1 \Rightarrow \sum a = \pm 1$$

$$\text{Now } \sum a^3 - 3abc = (\sum a)(\sum a^2 - \sum ab)$$

$$2 - 3abc = \pm 1(1 - 0)$$

$$2 - 3abc = \pm 1$$

$$\begin{array}{l} (+) \quad \quad \quad (-) \\ \swarrow \quad \quad \quad \searrow \\ \boxed{3abc = 1} \quad \boxed{3abc = 3} \\ \boxed{abc = \frac{1}{3}} \quad \boxed{abc = 1} \end{array}$$

Q.21 Let the position vectors of points 'A' and 'B' be $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda : 1 (\lambda > 0)$. If O is the origin and

$$\vec{OB} \cdot \vec{OP} - 3 \left| \vec{OA} \times \vec{OP} \right|^2 = 6, \text{ then } \lambda \text{ is equal to } \underline{\hspace{2cm}}$$

माना बिन्दुओं 'A' तथा 'B' के स्थिति सदिश क्रमशः $\hat{i} + \hat{j} + \hat{k}$ तथा $2\hat{i} + \hat{j} + 3\hat{k}$, हैं। एक बिन्दु 'P' रेखा खंड AB को अन्तः अनुपात

$\lambda : 1 (\lambda > 0)$ में विभाजित करता है। यदि O मूल बिन्दु है तथा $\vec{OB} \cdot \vec{OP} - 3 \left| \vec{OA} \times \vec{OP} \right|^2 = 6$ है, तो λ बराबर है _____

Sol. 0.8

$$\vec{OA} = \langle 1, 1, 1 \rangle, \vec{OB} = \langle 2, 1, 3 \rangle$$

$$\begin{array}{c} \lambda \quad \quad \quad 1 \\ \downarrow \quad \quad \quad \downarrow \\ \text{A} \quad \quad \quad \text{P} \quad \quad \quad \text{B} \end{array}$$

$$\vec{OP} = \left(\frac{2\lambda + 1}{\lambda + 1}, 1, \frac{3\lambda + 1}{\lambda + 1} \right)$$

$$\vec{OB} \cdot \vec{OP} = \frac{2(2\lambda + 1)}{\lambda + 1} + 1 + \frac{3(3\lambda + 1)}{\lambda + 1}$$

$$= \frac{14\lambda + 6}{\lambda + 1}$$

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$$\begin{aligned}
 |\overline{OA} \times \overline{OP}|^2 &= |\overline{OA}|^2 |\overline{OP}|^2 - (\overline{OA} \cdot \overline{OP})^2 \\
 3 \cdot \left(\frac{(2\lambda + 1)^2 + (\lambda + 1)^2 + (3\lambda + 1)^2}{(\lambda + 1)^2} \right) - \left(\frac{2\lambda + 1 + \lambda + 1 + 3\lambda + 1}{\lambda + 1} \right)^2 \\
 &= \frac{1}{(\lambda + 1)^2} \{ 3(14\lambda^2 + 12\lambda + 3) - (6\lambda + 3)^2 \} \\
 &= \frac{1}{(\lambda + 1)^2} \{ 6\lambda^2 \}
 \end{aligned}$$

$$\text{Now } \frac{14\lambda + 6}{\lambda + 1} - 3 \left(\frac{6\lambda^2}{(\lambda + 1)^2} \right) = 6$$

$$\begin{aligned}
 (14\lambda + 6)(\lambda + 1) - 18\lambda^2 &= 6(\lambda + 1)^2 \\
 -4\lambda^2 + 20\lambda + 6 &= 6\lambda^2 + 12\lambda + 6 \\
 10\lambda^2 - 8\lambda &= 0 \\
 \lambda(10\lambda - 8) &= 0
 \end{aligned}$$

$$\therefore \lambda > 0$$

$$\boxed{\lambda = .8}$$

Q.22 Let $[t]$ denote the greatest integer less than or equal to t . Then the value of

$$\int_1^2 |2x - [3x]| dx \text{ is } \underline{\hspace{2cm}}$$

यदि $[t]$ महत्तम पूर्णांक $\leq t$ है, तो $\int_1^2 |2x - [3x]| dx$ का मान बराबर है _____

Sol. 1

$$\begin{aligned}
 &\int_1^2 |2x - [3x]| dx \\
 &3x = t \\
 &= \frac{1}{3} \int_3^6 \left| \frac{2t}{3} - [t] \right| dt \\
 &= \frac{1}{9} \left[\int_3^6 |2t - 3[t]| dt \right] \\
 &= \frac{1}{9} \left[\int_3^4 |2t - 9| + \int_4^5 |2t - 12| + \int_5^6 |2t - 15| \right] dt \\
 &= \frac{1}{9} \left[\int_3^4 (9 - 2t) + \int_4^5 (12 - 2t) + \int_5^6 (15 - 2t) \right] dt
 \end{aligned}$$

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$$\begin{aligned} &= \frac{1}{9} [9 \cdot 1 + 12 \cdot 1 + 15 \cdot 1 - [4^2 - 3^2] - [5^2 - 4^2] - [6^2 - 5^2]] \\ &= \frac{1}{9} [36 - [4^2 - 3^2 + 5^2 - 4^2 + 6^2 - 5^2]] \\ &= \frac{1}{9} [36 - 36 + 9] = 1 \end{aligned}$$

Q.23 If $y = \sum_{k=1}^6 k \cos^{-1} \left\{ \frac{3}{5} \cos kx - \frac{4}{5} \sin kx \right\}$, then $\frac{dy}{dx}$ at $x=0$ is _____

यदि $y = \sum_{k=1}^6 k \cos^{-1} \left\{ \frac{3}{5} \cos kx - \frac{4}{5} \sin kx \right\}$, तो $x=0$ पर $\frac{dy}{dx}$ का मान है _____

Sol. 91

$$y = \sum_{k=1}^6 k \cos^{-1} \{ \cos(kx + \theta) \}$$

$$\text{where } \tan \theta = \frac{4}{3}$$

$$y = \cos^{-1}(\cos(x+\theta)) + 2\cos^{-1}(\cos(2x+\theta)) + \dots + 6\cos^{-1}(\cos(6x+\theta))$$

$$\left. \frac{dy}{dx} \right|_{x=0} = \frac{\sin(\theta)}{\sqrt{1 - \cos^2 \theta}} + \dots$$

$$= 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3 + \dots + 6 \cdot 6$$

$$= \sum 6^2 = \frac{6 \cdot 7 \cdot 13}{6} = 91$$

Q.24 If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, \dots, b_{11}$ is 90, then the common difference of this A.P. is _____

यदि $b_1, b_2, b_3, \dots, b_{11}$ एक वर्धमान A.P. है और इसके पदों का प्रसरण 90 है, तो इस A.P. का सार्व अन्तर है _____

Sol. 3

$$\text{Var}(x) = \frac{\sum bi^2}{11} - \left(\frac{\sum bi}{11} \right)^2$$

$$90 = \frac{a^2 + (a+d)^2 + (a+2d)^2 + \dots + (a+10d)^2}{11}$$

$$\left(\frac{a + a + d + a + 2d + \dots + (a + 10d)}{11} \right)^2$$

$$10890 = 11 \{ 11a^2 + 385d^2 + 110ad \} - \{ 11a + 55d \}^2$$

$$10890 = 1210d^2$$

$$d = 3$$

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Q.25 For a positive integer n , $\left(1 + \frac{1}{x}\right)^n$ is expanded in increasing powers of x . If three consecutive coefficients in this expansion are in the ratio, 2:5:12, then n is equal to _____

एक धन पूर्णांक n के लिए, $\left(1 + \frac{1}{x}\right)^n$ को x की बढ़ती घातों में प्रसारित किया गया है। यदि इस प्रसार में तीन क्रमागत गुणांकों का

अनुपात 2:5:12 है, तो n बराबर है _____

Sol. 118

Let 3 consecutive coH are

$${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} :: 2:5:12$$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{5} \quad \& \quad \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5}{12}$$

$$\frac{r}{n-r+1} = \frac{2}{5} \quad \& \quad \frac{r+1}{(n-r)} = \frac{5}{12}$$

$$7r = 2n + 2 \quad \& \quad 17r = 5n - 12$$

$$\Rightarrow \frac{2n+2}{7} = \frac{5n-12}{17}$$

$$= 34n + 34 = 35n - 84$$

$$\Rightarrow n = 118$$

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